A family of quantum interiors for ordinary and stringy black holes II: Geometric considerations and global properties

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Abstract

A family of spacetimes suitable for describing the matter conditions of a static, spherically symmetric quantum vacuum is studied, as well as its reliability for describing a regular model for the interior of a semiclassical, static black hole —without ever invoking a mass shell for the final object. In paper I, this condition was seen to limit the search to only one, distinguished family, that was investigated in detail. Here it will be proven that, aside from being mathematically generic (in its uniqueness), this family exhibits beautiful physical properties, that one would reasonably demand from a collapse process, including the remarkable result that isotropization may take place conveniently far from the (unavoidable) regularization scale. The analysis is also extended in order to include the possibility of a stringy core, always within the limits imposed by the semiclassical approach to gravitation. This constitutes a first approximation to the final goal of trying to characterize a regular, self-gravitating, stringy black hole.

1 Introduction

This is the second of a couple of works devoted to the study of the spacetime properties of regular interiors for non-rotating black holes. In the first one [1] (referred to as paper I in the sequel), we identified and studied in detail a generic set of spacetimes, called GNRSS spacetimes. We also recovered previous attempts to understand this issue, and extended them, including a starting protocol for the quantization of the sources that may generate these effects. However, a proof of their uniqueness for describing the physics involved was still lacking. It will be proven here that, besides having a beautiful characterization from the mathematical point of view (concerning, in particular, its universality and uniqueness, under generic conditions of clear physical meaning), the GNRSS spacetimes do fulfill all the energy-stress properties currently expected to be associated to quantum effects in strong fields affecting non-rotating collapsed bodies.

The present work is twofold. In the first part, after reviewing, in Sect. 2, the construction of the generic families of static, spherically symmetric, quantum vacuums (SSQV), in Sect. 3 the geometrical properties of the solutions will be investigated, as well as those corresponding to the isotropic vacuums, which might describe the core of the object owing to the dominance of vacuum polarization. In Sect. 4, we will deal with the delicate question of matching those interiors with an exterior black hole metric, without having to invoke singular massive shells or the like. We will see how this condition limits the search to only one family, which will be mathematically characterized in full, and that it turns out to be physically suitable for all our aims. In Sect. 4.3, we solve a model for a regular black hole that does not assume isotropization to occur at the origin. Eventually, we recover the results of I, also in this case, thus completing the scheme started there.

On the other hand, in Sect. 5, we turn to the issue of replacing the interior, or at least the core of the collapsed object by a stringy black hole. To that end, in Sect. 6, we write the junction conditions for two spherically symmetric spacetimes. In Sect. 7, we apply them to a supersymmetric stringy black hole, always within the limits imposed by the semiclassical approach to gravitation, in order to obtain, at least, a first order approximation to our ultimate goal, which is to deal with a (regular) self-gravitating stringy black hole. The paper ends with some conclusions.

2 The families describing spherically symmetric vacuums

If $T_{\alpha\beta}$ is the stress-momentum tensor of a physical system, the classical vacuum is defined by $T_{\alpha\beta} = 0$. Quantum vacuums are usually defined through a cosmological constant, Λ, e.g. $T_{\alpha\beta} = \Lambda g_{\alpha\beta}$, which yields, e.g., the de Sitter spacetimes. Because of various physical arguments (see e.g. [2]–[12]) it is expected that these spacetimes describe, in first approximation, the core of a collapsed object, i.e. a black hole. Nevertheless, a de Sitter spacetime cannot be matched with a black hole exterior solution. A straightforward way out consists in accepting the presence of singular mass shells [5, 6, 13], but this introduces too high arbitrariness, as mentioned elsewhere. A different solution is to extend the definition of quantum vacuum. For the problem we want to deal with, which exhibits spherical symmetry, one may define that a solution of Einstein's equations corresponds to a SSQV whenever $T_{\alpha\beta} \neq 0$ and $T^0_0 = T^1_1$, $T^2_2 = T^3_3$, for some orthonormal cobasis —that of a local observer. In this way, any local observer adapted to the spherical symmetry will measure exactly the same values of $T_{\alpha\beta}$ (see e.g. [7, 14]). For the moment, we shall only consider static vacuums, because the first aim will be to describe the spacetime structure of the interior of a regular black hole.

We shall now give the families of spacetimes that are suitable to become SSQVs. Any static spherically symmetric spacetime can be conveniently described by

$$ds^2 = -F(r)\,dt^2 + F^{-1}(r)\,dr^2 + G^2(r)\,d\Omega^2, \tag{1}$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta\,d\varphi^2$. There are certainly other ways to represent these spacetimes, which avoid the problems occurring near the possible horizons, or by putting $R^2 d\Omega^2$, provided $G' \equiv dG(r)/dr \neq 0$ (see e.g. [15, 16]). Later, we will deal with some of these possibilities, but here it will suffice to consider the former representation. A standard calculation of $T_{\alpha\beta}$ yields, for a local observer at rest with respect to the coordinate grid of (1),

$$\rho = \frac{1}{G^2}\Big[1 - F(G'^2 + 2GG'') - GG'F'\Big], \tag{2}$$

$$p = \frac{1}{G^2}\Big[-1 + FG'^2 + GG'F'\Big], \tag{3}$$

$$p_2 = p_3 = \frac{F''}{2} + \frac{FG''}{G} + \frac{F'G'}{G}. \tag{4}$$

Imposing $T^0_0 = T^1_1$, $T^2_2 = T^3_3$ requires studying the condition $\rho + p = 0$. This yields (for $G \neq 0$)

$$F\,G'' = 0. \tag{5}$$

If $F = 0$, the expression (1) is useless. In this case the spacetimes can be written as[1]

$$ds^2 = 2\,dT\,dr + 2\,dr^2 + G^2(r)\,d\Omega^2. \tag{6}$$

In the orthonormalized cobasis given by

$$\mathbf{\Theta}^0 = dT/\sqrt{2}, \quad \mathbf{\Theta}^1 = dT/\sqrt{2} + \sqrt{2}\,dr, \quad \mathbf{\Theta}^2 = G\,d\theta, \quad \mathbf{\Theta}^3 = G\sin\theta\,d\varphi, \tag{7}$$

the Ricci tensor takes the form

$$\begin{aligned}\mathbf{Ricci} = \; & \frac{G''}{G}(-\mathbf{\Theta}^0 \otimes \mathbf{\Theta}^0 + \mathbf{\Theta}^0 \otimes \mathbf{\Theta}^1 + \mathbf{\Theta}^1 \otimes \mathbf{\Theta}^0 - \mathbf{\Theta}^1 \otimes \mathbf{\Theta}^1) \\ & + \frac{1}{G^2}(\mathbf{\Theta}^2 \otimes \mathbf{\Theta}^2 + \mathbf{\Theta}^3 \otimes \mathbf{\Theta}^3).\end{aligned} \tag{8}$$

While, for a SSQV we must have (the conditions on $T_{\alpha\beta}$ being directly translated into conditions for $R_{\alpha\beta}$)

$$\mathbf{Ricci} = R_{00}(-\mathbf{\Theta}^0_N \otimes \mathbf{\Theta}^0_N + \mathbf{\Theta}^1_N \otimes \mathbf{\Theta}^1_N) + R_{22}(\mathbf{\Theta}^2_N \otimes \mathbf{\Theta}^2_N + \mathbf{\Theta}^3_N \otimes \mathbf{\Theta}^3_N), \tag{9}$$

where $\{\mathbf{\Theta}^\Omega_N\}$ is some orthonormalized cobasis, not necessarily coincident with the one used in the computation of (8). Therefore, we must look for an orthonormalized cobasis for which the Ricci tensor (8) becomes of the type (9). Clearly this is the same as finding out whether we can have linear expressions $\mathbf{\Theta}^0_N \equiv A\mathbf{\Theta}^0 + B\mathbf{\Theta}^1$, and $\mathbf{\Theta}^1_N = C\mathbf{\Theta}^0 + D\mathbf{\Theta}^1$, with $-\mathbf{\Theta}^0_N \cdot \mathbf{\Theta}^0_N = \mathbf{\Theta}^1_N \cdot \mathbf{\Theta}^1_N = \mathbf{\Theta}^0_N \cdot \mathbf{\Theta}^1_N + 1 = 1$. However, the form of the SSQV is invariant under these changes (for they are adapted, by definition, to the spherical symmetry). The only solution is

$$G'' = 0. \tag{10}$$

If $F \neq 0$, we also have $G'' = 0$. Thus $G'' = 0$ constitutes the proper characterization of any possibility.

From $G'' = 0$ two distinct alternatives appear

$$G = \gamma, \quad \text{or} \quad G = \alpha\,r + \gamma, \tag{11}$$

where $\alpha(\neq 0)$ and γ are constant. In order to include the possible horizons, we write the metrics (1) under the common form

$$ds^2 = -(1-H)\,dT^2 + 2H\,dT\,dr + (1+H)\,dr^2 + \gamma^2\,d\Omega^2, \tag{12}$$

$$ds^2 = -(1-H)\,dT^2 + 2H\,dT\,dr + (1+H)\,dr^2 + (\alpha r + \gamma)^2\,d\Omega^2, \tag{13}$$

[1] It is first necessary to change the coordinate system of (1) by $dT \equiv dt + (1-F)/F dr$ while keeping the rest unchanged. Then one can impose $F = 0$.

where $H \equiv 1 - F$, and the coordinate change has been given elsewhere. The second case is in fact equivalent to the case $\alpha = 1$, $\gamma = 0$, as we shall now prove, because we are dealing with families of spacetimes.

If we perform the coordinate change $dT = \alpha d\tilde{t} + (\alpha^2 - 1)dr$, $\tilde{r} \equiv \alpha r + \gamma$, leaving θ and φ unchanged, we get

$$\begin{aligned} ds^2 = \;& -\alpha^2(1-H)\, d\tilde{t}^2 + 2(\alpha^2 H + 1 - \alpha^2)\, d\tilde{t}\, d\tilde{r} + (2 - \alpha^2 + \alpha^2 H)\, d\tilde{r}^2 + \\ & \tilde{r}^2\, d\Omega^2 \end{aligned}$$

which shows that, by choosing $\tilde{H} \equiv \alpha^2 H + 1 - \alpha^2$, one has

$$ds^2 = -(1-\tilde{H})\, d\tilde{t}^2 + 2\tilde{H}\, d\tilde{t}\, d\tilde{r} + (1+\tilde{H})\, d\tilde{r}^2 + \tilde{r}^2\, d\Omega^2. \tag{14}$$

To summarize, there are *only* two —non-equivalent— families of SSQV. The case with $G' = 0$ is characteristic of the Nariai solution [17, 18]. The Nariai solution is a solution of Einstein's equations for the same pattern as the de Sitter solution, i.e. $T_{\alpha\beta} = \Lambda_0 g_{\alpha\beta}$, being Λ_0 the cosmological constant. The difference lies in the "radial" coordinate. In the Nariai case there is no proper center for the spherical symmetry. Therefore, we shall call the spacetimes with $G' = 0$ generalized Nariai metrics. Finally, the other case corresponds to the GNRSS spaces, already studied in I, which constitute a distinguished family of the class of Kerr-Schild metrics.

3 Geometrical properties of the SSQV

We denote by t, r, θ, φ the coordinates of the forms (12) and (14), since no confusion can arise in what follows.

3.1 Generalized Nariai metrics

Using an orthonormal cobasis defined as

$$\begin{aligned} &\mathbf{\Theta}^0 = \left(1 - \tfrac{H}{2}\right) dT - \tfrac{H}{2}\, dr, \quad \mathbf{\Theta}^1 = \left(1 + \tfrac{H}{2}\right) dr + \tfrac{H}{2}\, dT, \\ &\mathbf{\Theta}^2 = \gamma\, d\theta, \quad \mathbf{\Theta}^3 = \gamma\, \sin\theta\, d\varphi, \end{aligned} \tag{15}$$

we see that the Riemann tensor has as independent components ($A' \equiv dA/dr$)

$$R_{0101} = -H''/2, \quad R_{2323} = 1/\gamma^2. \tag{16}$$

The Ricci tensor is characterized by

$$R_{00} = -R_{11} = -H''/2, \quad R_{22} = R_{33} = 1/\gamma^2. \tag{17}$$

The scalar curvature is simply $R = H'' + 2/\gamma^2$, and the Einstein tensor has the following non-zero components

$$G_{00} = -G_{11} = 1/\gamma^2 \equiv \Lambda_0, \quad G_{22} = G_{33} = -H''/2, \tag{18}$$

where we have already identified the energy-matter density with the value of the cosmological constant, thanks to the presence of the Nariai solution inside this family. The isotropic solution is very important in order to set the type of spacetime to be chosen for the core of the object. An immediate calculation yields $H = \Lambda_0 r^2 + br + c$, where b and c are arbitrary constants. Without losing generality, we can set b, $c = 0$ (as they are clearly gauge freedoms for any spacetime in the family). Thus, the *only isotropic* quantum vacuum belonging to the family is the Nariai solution.

3.2 The GNRSS metrics

We refer the reader to I for details on the geometrical properties of these spacetimes. The main results: the isotropic GNRSS is the de Sitter solution, given by $H = (\Lambda_0/3)r^2$, and the exterior metric for a static black hole also belongs to the GNRSS family (e.g. the Reissner-Nordström one corresponds to $H = 2M/r - Q^2/r^2$, where M is the ADM mass and Q its electric charge.)

It is worth recalling that both these isotropic quantum vacuums are regular at the origin.[2]

4 The interior structure of a regular black hole

In this section, we introduce what may be viewed as a trial model for a regular black hole. We do not address here the question of relating each claimed spacetime solution with a (quantized) source origin. For now, we will only consider the main features of the desired structure.

For the exterior region, we can think of any (classical-dominated) black hole solution. For instance, the Schwarzschild, Reissner-Nordström, Kottler-Trefftz [18]–[20], or the like,

[2]The only singular "isotropic" solution belongs to the GNRSS family and is actually the Schwarzschild solution. It is nevertheless a true, or classical, vacuum solution.

solutions. The third is simply a Schwarzschild-de Sitter solution which would allow for the introduction of a non-zero value of the (true) cosmological constant. In fact, in I we added the quantum corrections to these exteriors coming from vacuum polarization, but these contributions are only relevant near the surface of the collapsed body and can be often dismissed. A remarkable property is that all these spacetimes belong to the GNRSS class[3] Then, for the interior of the body, we select a certain SSQV solution representing a transient state, relevant until the point when the spacetime becomes isotropized at the core of the object. The exterior solution and the core are almost fixed. On the contrary, the transient region is fairly free, as long as a study of the source origin is not carried out. The fact that Nariai, or de Sitter, and Schwarzschild solutions cannot be directly connected, forces one (as mentioned in the Introduction), to consider a smooth transition zone. This is dominated by the SSQV solution though, eventually, complete isotropization is expected to occur deep inside the body, owing to the dominance of vacuum polarization. If another type of effects dominate, as for instance strings, then the core could change. We now turn the attention towards the reliability of the two previous families as candidates for solving the present scheme.

4.1 The matching of generalized Nariai and GNRSS spacetimes

Assuming the exterior solution is actually a member of the GNRSS family, we shall first discuss if it is possible, for any such member, to match with some member of the generalized Nariai class. This will provide a direct check of the possibility for an eventual combination between the two.

The general form of a hypersurface that clearly adjusts itself to the spherical symmetry of any of these spacetimes is:

$$\Sigma : \begin{cases} \theta = \lambda_\theta, \\ \varphi = \lambda_\varphi, \\ r = r(\lambda), \\ t = t(\lambda), \end{cases} \tag{19}$$

where $\{\lambda, \lambda_\theta, \lambda_\varphi\}$ are the parameters of the hypersurface. Its corresponding tangent vectors are

$$\vec{e}_\theta = \partial_{\lambda_\theta} \stackrel{\Sigma}{=} \partial_\theta, \quad \vec{e}_\varphi = \partial_{\lambda_\varphi} \stackrel{\Sigma}{=} \partial_\varphi, \quad \vec{e}_\lambda = \partial_\lambda \stackrel{\Sigma}{=} \dot{r}\partial_r + \dot{t}\partial_t, \tag{20}$$

[3] They can be obtained by choosing $H(r) = 2M/rQ^2/r^2 + \Lambda_1 r^2$, where M, Q and Λ_1 are the mass, the charge and a external cosmological constant, respectively.

where the dot means "derivative with respect to λ". The normal one-form is then ($\mathbf{n}\cdot\vec{e}_i = 0$, $i = \{\theta, \varphi, \lambda\}$)

$$\mathbf{n} \stackrel{\Sigma}{=} \sigma(\dot{r}dt - \dot{t}dr), \tag{21}$$

If $\mathbf{n}$ is a null one-form, σ is a free function. Otherwise, $\mathbf{n}$ can be normalized, e.g. $\mathbf{n}\cdot\mathbf{n} = \pm 1$, and σ can be chosen to be

$$\sigma_\pm \stackrel{\Sigma}{=} \frac{\pm 1}{\sqrt{|\dot{t}^2 - \dot{r}^2 + H(\dot{r} - \dot{t})^2|}}. \tag{22}$$

The first junction conditions reduce to the coincidence of the first differential form of Σ at each spacetime. One must thus identify both hypersurfaces in some way. The identification of $(\lambda_i)_1$ with $(\lambda_i)_2$ (1 and 2 label each of the spacetimes) is clearly most natural, due to the symmetry of the above scheme. This yields (if 1 labels the GNRSS spacetime and 2 the generalized Nariai one)

$$r_1(\lambda) = \gamma = \text{const.}, \tag{23}$$

$$-\dot{t}_1^2(1 + H_1) = \dot{r}_2^2 - \dot{t}_2^2 + H_2(\dot{r}_2 + \dot{t}_2)^2. \tag{24}$$

The second set of junction conditions comes from (we do not use any singular mass shell, albeit this could be added without problem)

$$\left[\mathcal{H}_{ij}\right] = 0, \tag{25}$$

where $\mathcal{H}_{ij}$ is defined by

$$\mathcal{H}_{ij} \stackrel{\Sigma}{\equiv} -m_\rho\left(\frac{\partial^2\phi^\rho}{\partial\lambda^i\partial\lambda^j} + \Gamma^\rho_{\mu\nu}\frac{\partial\phi^\mu}{\partial\lambda^i}\frac{\partial\phi^\nu}{\partial\lambda^j}\right). \tag{26}$$

Here $\vec{m}$ is any vector that completes the set $\{\vec{e}_i\}$ to form a vectorial basis of the manifold[4] (see e.g. [21] and I). We will select, for the GNRSS spacetime,

$$\vec{m}_1 = \dot{r}_1\partial_{t_1} - \dot{t}_1\partial_{r_1}.$$

This choice has the property that $\vec{m}_1 \cdot \mathbf{n}_1 = \sigma_1(\dot{r}_1^2 + \dot{t}_1^2)$. It is a convenient one because, if it vanishes, the hypersurface Σ becomes degenerate and the joining process itself cannot be carried out. For the Nariai spacetimes, as there is no preferred radial coordinate to be

[4]For the cases when $\mathbf{n}$ is non-null, $\vec{m}$ can be chosen simply as $\vec{n}$, and $\mathcal{H}_{ij}$ becomes the second fundamental form. $\mathcal{H}_{ij}$ allows for dealing with a transition at the event horizon of the black hole.

identified with r_1, we have to leave $\vec{m}_2$ free, and see if there is some choice that makes the matching possible. Of course, it must at least satisfy $\vec{m} \cdot \mathbf{n}_2 \neq 0$, if Σ is a hypersurface. Furthermore, $\phi^\rho(\lambda)$ are the parametric equations of the hypersurface ($\{\phi^0, \phi^1, \phi^2, \phi^3\} = \{t, r, \theta, \varphi\}_\Sigma$), and $\Gamma^\rho_{\mu\nu}$ the connection coefficients.

Eqs. (25) yields, for *any* $\vec{m}_2$,

$$r_1 \dot{t}_1 = 0, \tag{27}$$

An immediate consequence of (27) is that the matching between any generalized Nariai spacetime and any member of the GNRSS is *not* possible (if $\dot{t}_1 = 0$, since also $\dot{r}_1 = 0$, the matching becomes meaningless, i.e. it is only valid for an "instant" and for a specific "place".) The only way to overcome this impossibility is calling for singular mass shells. However our aim is to avoid such unphysical situation. We do not mean by this that the transition zone cannot become small in comparison with the isotropic region, but that we disregard exact singular mass shells as physical solutions for the structure of the collapsed body because it would affect even to the claimed smoothness of the model (see also the conclusions in [13]).

To summarize, if vacuum polarization is to be the dominant quantum effect, the most simple way to construct a regular black hole is to build it upon GNRSS spacetimes .

4.2 The matching of two GNRSS spacetimes

We refer the reader to I, where explicit expressions for the matching conditions are given. It follows from them, that the only physically acceptable solutions are the hypersurfaces defined by (now 1 and 2 each label a GNRSS space) $r_1 = r_2 = R =$ const. The rest of the conditions, Eqs. (24), (25), are rather simple, namely,

$$H_1|_{r_1=R} = H_2|_{r_2=R}, \quad H_1'|_{r_1=R} = H_2'|_{r_2=R}, \quad \dot{t}_1 = \dot{t}_2, \tag{28}$$

where $H'|_{r=R} \equiv dH(r)/dr|_{r=R}$. This also shows that the coordinate system chosen in (14) is in fact a privileged one, in which the metric takes an explicit C^1 form.

Thus, the most plausible scheme is the one depicted in Tab. 1. Obviously, the properties of the intermediate interior region are most important, the rest already having a clear interpretation in physical terms, including quantum fields that may act as sources for the core, i.e. de Sitter spacetime.

Core	**Interior**	**Exterior**	∞
de Sitter	GNRSS	Non – Rotating Black Hole	Cosmological Term

Table 1: Physically meaningful scheme for a static regular black hole interior dominated by quantum vacuum polarization. The cosmological term is optional.

Interior	**Exterior**	∞
GNRSS	Non – Rotating Black Hole	Cosmological Term

Table 2: Simplified picture corresponding to Tab. 1 where $\text{GNRSS}_{int} \to$ de Sitter, for $r \to 0$.

A natural simplification of the scheme above is to demand that the interior GNRSS solution tends to the de Sitter solution as $r \to 0$. In that case, we get the simplified picture of Tab. 2. This is a common assumption to be found in the literature (see e.g. [5]–[8], [22]–[24]). It was also made in I.

We shall now study the case of Tab. 1 in detail because it is free from the drawback of demanding $r \to 0$, i.e. the imposition that the spacetime should be extended to regions where quantum effects play a dominant role, and where the classical notions of space and time cease to be valid, together with the existence of an inner Cauchy horizon (see e.g. [23]), which will lie now very far from the scheme depicted in Tab. 1, as we will show. However the second scheme actually allows for a complete, and easier, study of the semiclassical zone, as shown in paper I.

4.3 Constraints for the de Sitter-GNRSS model

Looking at Tab. 1 and noticing that the core and the exterior parts of the model only depend on Λ_0, M, Q, it turns out that two sets of constraints for the unknown function H_{int} appear. Of course, the parameters of the core, i.e. Λ_0, and those coming from the exterior solution, i.e. the mass, the charge, and the (exterior) cosmological constant, also impose some restrictions, but we will consider them as free data.

The matching of the de Sitter and the interior GNRSS yields

$$\Sigma_{\text{dS}} : r_{\text{dS}} = r_{int} \equiv R_{\text{dS}} = \text{const.}, \quad (\Lambda_0/3)R_{\text{dS}}^2 = H_{int}(R_{\text{dS}}), \\ (2\Lambda_0/3)R_{\text{dS}} = H'_{int}(R_{\text{dS}}), \tag{29}$$

where $R_{\rm dS}$ may be interpreted as the "de Sitter radius" of the object, the scale where isotropization takes place. Analogously, the matching of the interior and the exterior GNRSS gives

$$\Sigma_{\rm body} : \tilde{r}_{\rm body} = r_{\rm body} \equiv R = \text{const.}, \quad H_{int}(R) = H_{ext}(R), \\ H'_{int}(R) = H'_{ext}(R), \tag{30}$$

where we have set $\tilde{r}$ for the interior region simply because it is different from $R_{\rm dS}$ (in fact, only $R_{\rm dS} < R$ makes sense).

We thus have two conditions at $R_{\rm dS}$ and two more at R. For the scheme 2 we have two conditions at $r \to 0$ and two more at R. Both schemes are then similar, but now we have a new unknown, namely, $R_{\rm dS}$. In order to see the changes, it is worth solving a particular set of models. We will consider the analogous of the two-power models of I. In those, $H_{int}(r) = a_p r^p + a_q r^q$, where p, q are real numbers (obviously $p < q$ would suffice). It is worth noticing that the case $p = q$ is incompatible with the conditions (29), (30), e.g. the de Sitter spacetime alone is not sufficient to fulfill the internal structure of the black hole.

For the exterior solution we shall choose a "Schwarzschild-de Sitter" model which accounts for current astrophysical-cosmological observations on black holes (see e.g. [25]–[27]), whereas for the core we use a de Sitter solution. The functions H are, respectively, $H_{ext} = 2m/r + (\Lambda_1/3)r^2$, and $H_{\rm core} = (\Lambda_2/3)r^2$, where Λ_1, Λ_2 are the cosmological terms of the exterior region and the core, respectively (one expects $\Lambda_1 << \Lambda_2$). Isolating the coefficients a_p, a_q from Eqs. (29), (30), using the previous expressions for H_{ext} and H_{int}, we get (remember $p \neq q$)

$$a_p = \left(\frac{q-2}{q-p}\right)\left(\frac{\Lambda_2}{3R_{\rm dS}^{p-2}}\right) = \left(\frac{1+q}{q-p}\right)\left(\frac{2m}{R^{q+1}}\right) - \left(\frac{2-q}{q-p}\right)\left(\frac{\Lambda_1}{3R^{p-2}}\right), \tag{31}$$

and an analogous result for a_q, interchanging p and q. If $q = 2$, one gets immediately $m = 0$. Therefore $p, q \neq 2$. We do not recover now the lowest power model of paper I, because the body suddenly becomes isotropized at $r = R_{\rm dS}$. Moreover, if $p = -1$, necessarily $q = -1$. Therefore $p = q$ and there is no solution of (29), (30). The constraints are then ($p \neq q$, $p, q \neq -1, 2$)

$$\frac{\Lambda_2}{3} = \left(\frac{q+1}{q-2}\right)\left(\frac{2m}{RR_{\rm dS}^2}\right)\left(\frac{R_{\rm dS}}{R}\right)^p + \frac{\Lambda_1}{3}\left(\frac{R_{\rm dS}}{R}\right)^{p-2} \\ = \left(\frac{p+1}{p-2}\right)\left(\frac{2m}{RR_{\rm dS}^2}\right)\left(\frac{R_{\rm dS}}{R}\right)^q + \frac{\Lambda_1}{3}\left(\frac{R_{\rm dS}}{R}\right)^{q-2}. \tag{32}$$

From the expressions above, we realize that $R_{\rm dS}$ and R can be obtained in terms of m, Λ_1, Λ_2, p and q. However, in order to arrive to explicit algebraical relations, we will disregard

the contribution of Λ_1. Another reason to do so comes from observational arguments. We expect $\Lambda_2 >> \Lambda_1$. Furthermore, we also expect $R_{dS} < R$, $R \leq 2m$ (the collapsed object is at, or beyond, the event horizon) so that $\Lambda_1 R^2 << 1$ and the contribution of the terms with Λ_1 in (32) are completely negligible in front of the rest. We then get

$$R = \left(\frac{6m}{R_{\rm dS}^2 \Lambda_2}\right)\left[\left(\frac{p-2}{p+1}\right)^p \left(\frac{q+1}{q-2}\right)^q\right]^{\frac{1}{q-p}}. \tag{33}$$

If we define (as in I, Λ_1 neglected), $R_Q \equiv \sqrt[3]{6m/\Lambda_2}$, we can rewrite (33) as

$$R = \left(\frac{R_Q^3}{R_{\rm dS}^2}\right)\left[\left(\frac{p-2}{p+1}\right)^p \left(\frac{q+1}{q-2}\right)^q\right]^{\frac{1}{q-p}}. \tag{34}$$

Setting $R \equiv 10^\beta R_Q$, $R_{\rm dS} \equiv 10^{-\gamma} R_Q$, we get

$$\beta = \frac{(q-2)\log|(q+1)/(q-2)| + (p-2)\log|(p-2)/(p+1)|}{3(q-p)}, \tag{35}$$

$$\gamma = \frac{(q+1)\log|(q-2)/(q+1)| + (p+1)\log|(p+1)/(p-2)|}{3(q-p)}. \tag{36}$$

One readily sees that $\beta + \gamma > 0$ for most situations. It is unnecessary to carry out a complete study of the exponents. Our aim is just to show the main features of the internal black hole structure by means of this simple (but already non-trivial) model. Some results are displayed in Table 3, for a number of different exponents. It is shown there that, regardless of the powers, the results are very similar, and equivalent to those obtained in paper I. Thus with high confidence the whole picture emerges as a likely structure for static black holes. One should also notice that, here, the problems of ascribing a de Sitter core when $r \to 0$ is solved, although, numerically, R does scarcely change with respect to the results of the simplified model in I. The proximity of the de Sitter region to the limiting hypersurface of the body has also been claimed as convenient in [10], where the Kerr black hole is studied, and justifies the idea of effective mass shells, though only in an *effective* sense, i.e. not arriving to the creation of a singular distribution.

Finally let us add that R_Q is obtained when one tries to match the de Sitter core with an exterior, Schwarzschild black hole metric, imposing only the natural condition $H_{\rm dS}(R) = H_{\rm Schw}(R)$. We already knew that this direct junction was impossible, unless mass shells are introduced. However, it is noticeable that the relevant order of magnitude is in complete agreement with that of the rigorous result.

	$(1000,3)$	$(3,100)$	$(3,4)$	$(-3,-2)$	$(-100,-4)$	$(-5000,-1000)$
$R_{\rm dS}$	0.999	0.99	0.4	0.86	0.996	$1-10^{-7}$
R	1.001	1.01	0.9	1.36	1.003	$1+10^{-7}$

Table 3: Values for $R_{\rm dS}$, R ($R_{\rm dS} < R$) in units of R_Q for several powers. In all cases, the values are very close to each other. Moreover, R_Q is very far away from the scale of regularizaton for any astrophysical, or more massive, object. Therefore they all lie in the semiclassical regime.

5 Extension to other possibilities for interiors

In this section, we turn our attention to other possible candidates for the interior of a black hole. In the previous ones, the models were built upon the idea that vacuum polarization or supergravity domain wall [10]–[12] would play an essential role. But it is also interesting to check whether a stringy black hole could actually describe the core of a black hole or, even, its transitory region. There are many results that point towards a correspondence between semiclassical black holes and stringy ones [28]–[32], [10]. Among the most widely studied solutions there are the supersymmetric ones. However, for our aim of considering a self-gravitating black hole, an approximate solution will suffice, provided the transition takes place at the semiclassical level. As with the previous models, this fact can be eventually justified by carrying out, *a posteriori*, a numerical analysis of the solutions obtained, (see paper I). We just impose here the most elementary conditions that are able to produce a stringy interior in the black hole, and we will leave to further work the calculation of the self-gravitating string (this issue being still under study, see e.g. [31]).

Stringy black holes do not correspond to SSQV solutions in general. Thus, we need to perform their matching with a GNRSS solution. However it seems more natural to carry out such a calculation in general. Namely, to find the conditions for two static spherically symmetric spacetimes to match with each other. In fact, this is a well-known program, specially if one does not include the possibility of null hypersurfaces, see e. g. [33, 34]. For the case of null hypersurfaces one can use the formalism in [35], allowing a transition at a horizon. The addition of null hypersurfaces could be of interest in those attempts at studying radiative processes in the interior region of the black hole, [36]–[38]. However, it would be a nuisance to have to alternate all the time between two different pictures, depending on which process we would like to study. It is possible and more interesting to perform a general study which can take into account different behaviors of the matter and radiation

in the black hole interior. Thus, we will consider a general hypersurface which may turn out to be null at any time or spatial point of the interior region. The necessary formalism to deal with this situation can be found in [21]. The general solution to this question is to be found in [23]. However the representation of the spacetimes being used is not a common one for the spacetime metrics we shall deal with, and the translation of those results to each of our cases turns out to be even more tedious than a direct computation. Thus, we shall follow here a different approach. It is important to note that, once the conditions are obtained, the process of matching itself becomes irrelevant. Therefore, we shall try to obtain the conditions as directly as possible.

6 The matching of static spherically symmetric spacetimes

Our aim here is to match two spacetimes that share the existence of an integrable Killing field and spherical symmetry. In order to get the most natural junction, we need to take profit of both symmetries exhaustively. The spherical symmetry is easy to identify in both spacetimes. The metric can always be written, for any of them, as

$$ds^2 = g_{AB}(R)\, dx^A\, dx^B + G^2(R)\, d\Omega^2, \tag{37}$$

where $A,\, B = T, R$; ∂_T has been chosen to be the integrable Killing vector and $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$. Moreover, if $G'(R) = 0$, we know that the spacetimes belong to the generalized Nariai family, in which case they only match with another member of its own family, as is easy to show. Thus, we will only deal with the situation $G'(R) \neq 0$. In this case, a direct redefinition of the R coordinate allows us to write

$$ds^2 = g_{AB}(r)\, dx^A\, dx^B + r^2\, d\Omega^2, \tag{38}$$

where $A,\, B = T, r$.

Spherical symmetry has thus been completely used. We now extract consequences from the presence of ∂_T, an integrable Killing field. The natural thing to do is to identify both vector fields, i.e. $\partial_{T_1} \overset{\Sigma}{\equiv} \partial_{T_2}$. However this is not a right choice, in general. If a Killing vector is multiplied by a constant factor, the resulting vector field is obviously a Killing vector field. Therefore, normalizing each Killing vector, when possible, gives the natural way to identify them. This is implemented in the junction process, if the hypersurfaces are spacelike or timelike everywhere, where the calculations are easier as well (see the previous footnote).

On the contrary, in the general case, we cannot rely on such normalization, and $\vec{m}$ takes the role of $\mathbf{n}$. But $\vec{m}$ is an extrinsic object with respect to the hypersurface. Thus, one has to take care of the identification process. The results do not depend on $\vec{m}$, provided the identification is the one desired. In Sect. 4 this was readily implemented. Let us show how it can be done now.

Any metric of interest (to our purpose) can be written as (recall the coordinate change to obtain (6), setting now $F = 1 - H$)

$$ds^2 = -(1 - H)\, dT^2 + \frac{2H}{g}\, dT\, dr + \frac{1 + H}{g^2}\, dr^2 + r^2\, d\Omega^2, \tag{39}$$

where H, $g \neq 0$ are functions of r only. Looking back to expressions (13), (14) and to their equivalence, we will put now $dT = g_0 dt + (g_0 - g_0^{-1})dr$, where g_0 is a constant, that will be related with the function g, as we shall see in a moment. With this coordinate change the metric takes the form[5]

$$ds^2 = -g_0^2(1 - H)\, dt^2 + 2\tilde{G}\, dt\, dr + \tilde{F}\, dr^2 + r^2\, d\Omega^2, \tag{40}$$

where $\tilde{G} = g_0^2(H - 1) + 1 + H(g_0 - g)/g$, and $\tilde{F} = 2 + g_0^2(H - 1) + (g_0 - g)[2H/g + 2/g_0^2 g + (g_0 - g)(H + 1)/g_0^2 g]$.

The junction conditions are then

$$[r] = 0, \quad [\dot{t}] = 0 \tag{41}$$

$$[\tilde{H}]\dot{t}^2 + 2[\tilde{G}]\dot{t}\dot{r} + \tilde{F}\dot{r}^2 = 0, \tag{42}$$

$$[\tilde{F}]\dot{t}\ddot{r} - [\tilde{G}](\ddot{t}\dot{t} - \ddot{r}\dot{r}) + [\tilde{H}]\dot{r}\ddot{t} + [\tilde{G}']\dot{r}^3 + [\tilde{H}' - (\tilde{F}'/2)]\dot{r}^2\dot{t} - [\tilde{H}']\dot{t}^3/2 = 0, \tag{43}$$

where $[f] \stackrel{\Sigma}{\equiv} f_2 - f_1$, and where we have put $\tilde{H} \equiv g_0^2(H - 1) + 1$. In (42) and (43) $\dot{t}$ and $\dot{r}$ are either $\dot{t}_1$, $\dot{r}_1$ or $\dot{t}_2$, $\dot{r}_2$, and $A' \equiv dA(r)/dr|_{r=r(\lambda)}$. The same conditions lead, in general, to a second order ordinary differential equation for r. In principle there is the possibility for asymptotic stopping solutions, i.e. solutions for which $r \to$ const. as $t \to \infty$, and also for null ones. A complete study of these possibilities is worth pursuing, after having decided which are the specific spacetimes to be considered in accordance with our purposes, i.e. provided some knowledge about $H(r)$, $g(r)$ is available. A special case is of great interest, since it constitutes the solution towards which any transitory solution should converge, in order to

[5] It is also equivalent to leave the T coordinate unchanged, and change $\vec{m}$. However, we prefer the characterization above, because it directly tells us which is the t coordinate to be identified in both spacetimes.

have a regular black hole interior, namely $r_1 = r_2 = R =$ const.. Under this restriction, the conditions become, simply,

$$[\dot{t}] = 0, \quad [\tilde{H}] = 0, \tag{44}$$

and

$$2[\tilde{G}]\ddot{t} - [\tilde{H}']\dot{t}^2 = 0, \tag{45}$$

where t is either t_1 or t_2. Choosing g_0 as g_Σ one gets $[\tilde{G}] = 0$ (the same result comes out directly in the case when the normal vector of Σ is non-null). The last conditions become then $[\tilde{H}'] = 0$. Thus, we arrive at the conclusion that the conditions emerging from the matching of two spherically symmetric spacetimes with an integrable Killing vector field are, for the case $r = R =$ const. and taking the maximum identification between them,

$$[\tilde{H}] = 0, \quad [\tilde{H}'] = 0, \tag{46}$$

where $\tilde{H} \equiv g_\Sigma^2(H-1) + 1$. An intrinsic characterization, valid for any representation of the form (37) or (38) (those are expressions that are often dealt with) is $\tilde{H} \equiv -g_\Sigma(\vec{\xi} \cdot \vec{\xi}) + 1$, $g_\Sigma \equiv [G'/|\det(g_{AB})|^{1/2}]_{r=R}$, where $\vec{\xi}$ is the Killing vector associated with the staticity of the solution (in some regions) of (37) or (38). It is not difficult to realize that the first condition on $\tilde{H}$ is nothing but the requirement of the mass function to be continuous across the hypersurface, while the second one is related with the continuity of the radial stress, or pressure, see e.g. (3).

7 An application to supersymmetric stringy black holes

The semiclassical expressions for supersymmetric stringy black holes are well-established (see e.g. [29, 30] and references therein). There are also other objects of interest, such as black strings, higher dimensional black holes, etc. In all cases one looks for a correspondence principle with general relativistic black holes. This equivalence, or transition, is usually reflected in the strength of the coupling constant, or the entropy (see e.g. [29]–[32] and references therein). Here we take a different viewpoint, coming from the above scheme, which turns out to be valid because all the fields are in the semiclassical domain. Thus, it would be interesting to see whether both approaches complement each other, or which are the restrictions that may be induced from the current scheme. The most interesting case

is that of a self-gravitating string (see e.g. [31, 32]). However the necessary ingredients —specially the corresponding spacetime metric— in order to tackle this problem are still under study. Here we will deal with the most simple (and widely considered) case only, i.e. that of a supersymmetric back hole.

A family of such black holes, related with electrically charged black holes, is given by (see [29, 30] for details)

$$ds^2 = -f^{-1/2}(r)\Big(1-\frac{r_0}{r}\Big)\,dt^2 + f^{1/2}(r)\Big[\Big(1-\frac{r_0}{r}\Big)^{-1}dr^2 + r^2\,d\Omega^2\Big], \tag{47}$$

where $f(r) = \prod_{i=1}^{4}[1+(r_0\sinh^2\alpha_i/r)]$, and where the α_i are related with the integer charges of the D-branes being used. If the correspondence occurs at a constant value of r, we get

$$r_1 + r_0\sinh^2\alpha = r_2 f_2^{1/4}(r_2) \equiv R = \text{const.} \tag{48}$$

$$\frac{2m}{R} - \frac{Q^2}{R^2} = 1 + \left[\Big(\frac{r_0}{r}-1\Big)\Big(1+\frac{rf'}{4f}\Big)^2\right]_{\Sigma_2}, \tag{49}$$

$$-\frac{2m}{R^2} + \frac{2Q^2}{R^3} = \left\{\Big(1+\frac{rf'}{4f}\Big)^2\left[\frac{f'}{2f}\Big(1-\frac{r_0}{r}\Big) - 2\frac{r_0}{r^2}\right]\right\}_{\Sigma_2}, \tag{50}$$

where we have used $g_\Sigma = G'|_{r=G^{-1}(R)}$, $G(r) = rf^{1/4}(r)$, and $\vec{\xi}\cdot\vec{\xi} = -f^{-1/2}(1-r/r_0)$. The subscript Σ_2 means that all these quantities refer to the interior region, to be evaluated at $r = r_2$. For the exterior metric, we have put $\alpha_i = \alpha_j \equiv \alpha$, for all i, j, because the exterior metric is that of a Reissner-Nordström black hole, for which

$$\begin{aligned} &2m = r_0\cosh 2\alpha, \quad Q^2 = r_0^2\sinh^2\alpha\,\cosh^2\alpha, \\ &r_0 = 2\sqrt{m^2-Q^2}, \quad 2\sinh^2\alpha = -1 + m/\sqrt{m^2-Q^2}, \end{aligned} \tag{51}$$

where m is the (ADM) mass of the black hole and Q is its electric change. Since $f_2(r_2) = \prod_{i=1}^{4}[1+(r_0\sinh^2\alpha_i/r)]_{\Sigma_2}$, the above conditions yield R as a function of six of the seven parameters, $M, Q, (r_0)_2, \alpha_i$. Detailed analysis shows that these conditions are easily fulfilled when $r_0 \to 0$, $\alpha_i \to \pm\infty$, with $r_0\sinh^2\alpha_i$ fixed. The resulting R is very close to $R_0 \equiv m+\sqrt{m^2-Q^2}$, i.e. the event horizon of the black hole. It is interesting to notice that $rf^{1/4}(r)$ is the radial coordinate, which has a direct interpretation in terms of the "size" of the object, and not of r alone. All this being in complete agreement with the expected transitions for extreme, and nearly extreme, supersymmetric black holes. The same idea could be extended to self-gravitating strings, where the physical scheme becomes more interesting for the higher plausibility of having regular interiors in this case. For instance, the expected order of magnitude of R, [31], should be recovered. This issue will be the matter of subsequent research.

8 Conclusions

In this work we have investigated, under quite general conditions, the question of using Einstein's theory of gravitation —extended to include semiclassical effects— with the purpose to constraint the physical structure of the emerging spacetime solutions that might be suitable for the description of the interiors of non-rotating black holes.

In the first part of the work we have made extensive use of general ideas, coming from plausible, expected quantum contributions to the energy-momentum tensor, in order to get the main features of the resulting models in terms of a spacetime viewpoint. For instance, we have exploited the idea that vacuum Polarization or supergravity, e.g. domain wall, may indeed play an essential role in the interior region. We have obtained the result that only two families fulfill the imposed requirement and, moreover, we have shown that *only one* of them is suitable for representing black hole interiors, what is certainly a most remarkable result. Moreover, we have extended the models of paper I in order to solve the problem of demanding *isotropization* near the regularization scale and have proven that this is indeed possible, with little changes in the previous results.

Then we have turned our attention to stringy black holes. Since the solutions we are interested in —self-gravitating strings— still need to be studied in more detail, we have just started this program by first giving the general conditions to be fulfilled by any spacetime with spherical symmetry and with some static region. Finally, we have applied the results obtained to a supersymmetric black hole, as a preliminar case. The result is that the proposed models are indeed generically compatible, specially concerning the extreme limit. This last situation is precisely the same for which the correspondence between semiclassical black holes and stringy ones has been recently confirmed in the literature (see e.g. [31, 32]).

Our overall conclusion is the following. In the first place, the results in paper I have opened a new window for the search of a compatible quantum field that, once regularized, may yield the same result for, at least, a particular energy-momentum tensor inside the general family of models considered. Second, once a corresponding Einsteinian metric associated with a quantum model is known, the scheme developed here might be certainly well suited to check the consistency of the involved physical parameters and even, in some cases, to assign explicit values to them. All these results, as a whole, compel us to believe that black hole singularities are likely to be removed by quantum effects, at least in the non-rotating case. [6]

[6] The rotating case, which is of major astrophysical interest, and the rotating and electrically charged one,

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which may be associated with spinning particles, seem to yield results very similar to the ones presented here, see e.g. [10]. This is also the outcome of preliminary calculations of ours, to be reported elsewhere [39].

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